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DIBARYONS AS AXIALLY SYMMETRIC SKYRMIONS

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ABSTRACT

Dibaryons configurations are studied in the framework of the bound state soliton model. A generalized axially symmetric ansatz is used to determine the soliton background. We show that once the constraints imposed by the symmetries of the lowest energy torus configuration are satisfied all spurious states are removed from the dibaryon spectrum. In particular, we show that the lowest allowed state in the $S = -2$ channel carries the quantum numbers of the H particle. We find that, within our approximations, this particle is slightly bound in the model. We discuss, however, that vacuum effects neglected in the present calculation are very likely to unbind the H.

1 Introduction

Since it was first proposed by Jaffe [1] that some hexa-quark state could be stable against strong decays, great attention has been devoted, both theoretically and experimentally, to this issue. Jaffe's suggestion was based on a bag model calculation where he showed that color-magnetic interactions favor the existence of a stable flavour singlet $S = -2$ state (the so–called H-dibaryon). Later on, however, it was shown that the inclusion of effects neglected in Ref.[1] (like i.e. symmetry breaking effects, center of mass corrections, pion cloud around the bag, etc.) tend to decrease the binding in a significant way [2], rendering it rather uncertain. Moreover, as recently discussed in Ref.[3], bag model predictions seem to be very sensitive to the bag constant which is not strongly constrained by empirical data. From the experimental point of view the situation is also unclear. Although some H weak decay events have been reported some time ago [4], other recent analysis provides no indication of a stable H–dibaryon [5]. New experiments are at the moment being carried out to investigate this issue further (see i.e. Ref.[6]).

Theoretical calculations have also been performed using various other models, like i.e. lattice QCD, non–relativistic quark model and soliton models (for a rather extensive list of references, see Refs.[5, 6]). Again, results have not been conclusive. In the case of the soliton models, most of the studies have been done using the collective coordinate $SU(3)$ (see Refs.[7, 8] and references therein) extensions of the Skyrme model. In this paper we will use an alternative method based on the bound state approach[9]. Since it has been recently shown [10, 11] that within this scheme hyperon properties are remarkably well described, this will provide another interesting insight into the problem of the strange dibaryons stability. A previous attempt to investigate the structure of dibaryons within the bound state approach was done in Ref.[12] where the H was found to be unbound. In that work, the simplified axially symmetric ansatz proposed in Ref.[13] was used for the skyrmion field. Indeed, such a simplified ansatz predicts that the $B = 2$ soliton mass is twice larger than $M_{sol}(B = 1)$ and is unstable. Here, we will use the improved axially symmetric ansatz proposed in Ref.[14]. Although this ansatz does not correspond exactly to the lowest axially symmetric energy configuration, diskyrmion properties (like i.e. soliton mass, rotational energies, etc.) computed with it turn out to be very similar to those obtained with the lowest energy torus configuration numerically found in Ref.[15].

An intriguing point discussed in Ref.[12] is the existence in the bound state soliton model of states which are forbidden in the quark model. In fact, a similar situation was found in Refs.[13, 14] for the case of non–strange dibaryons. In this paper we will show, however, that all these spurious states are removed from the spectrum once the constrains imposed by the symmetries of the problem are correctly taken into account. In particular, in our scheme the lowest $S = -2$ allowed dibaryon state is the flavour singlet, in agreement with the quark model prediction.

This paper is organized as follows. In Sec.2, we introduce the bound state model for arbitrary baryon number based on the improved axially symmetric ansatz. In Sec.3, we show how to construct dibaryon wavefunctions which are consistent with all the symmetries of the system. In Sec.4, numerical results are presented and discussed. In Sec.5,

conclusions are given. In Appendix A, we review the $SU(2)$ sector of the model. Finally, in Appendix B the quantization rules and the dibaryon quantum numbers are discussed in detail.

2 The Model

We start with the effective action for the simple Skyrme model with an appropriate symmetry breaking term, expressed in terms of the $SU(3)$–valued chiral field $U(x)$ as

$$\Gamma = \int d^4x \, \left\{ \frac{F_\pi^2}{16} \operatorname{Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] + \frac{1}{32e^2} \operatorname{Tr}\left[\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2\right] \right\} + \Gamma_{WZ} + \Gamma_{SB}, \tag{1}$$

where F_π is the pion decay constant ($= 186\ MeV$ empirically) and e is the so–called Skyrme parameter. In Eq.(1) Γ_{SB} is responsible for the explicit breaking of chiral symmetry. We use the following form for Γ_{SB}:

$$\begin{aligned}\Gamma_{SB} \; = \; \int d^4x \Bigg\{ & \frac{F_\pi^2 m_\pi^2 + 2F_K^2 m_K^2}{48} \operatorname{Tr}\left[U + U^\dagger - 2\right] + \frac{F_\pi^2 m_\pi^2 - F_K^2 m_K^2}{24} \operatorname{Tr}\left[\sqrt{3}\lambda^8 \left(U + U^\dagger\right)\right] \\ & + \frac{F_K^2 - F_\pi^2}{48} \operatorname{Tr}\left[\left(1 - \sqrt{3}\lambda^8\right)\left(U \partial_\mu U^\dagger \partial^\mu U + U^\dagger \partial_\mu U \partial^\mu U^\dagger\right)\right] \Bigg\} \ ,\end{aligned} \tag{2}$$

where λ^8 is the eighth Gell-Mann matrix and m_π and m_K represent the pion and kaon masses respectively and F_K is the kaon decay constant ($= 1.22\ F_\pi$). Eq.(2) accounts not only for the finite mass of the pseudoscalar mesons but also for the empirical difference between their decay constants. In previous calculations [9, 12] the kaon was found to be overbound to the soliton. It was recently shown [10] that this defect can be mostly eliminated if the difference in the decay constants is properly taken into account, as done in Eq.(2). Finally, Γ_{WZ} is the Wess–Zumino action,

$$\Gamma_{WZ} \; = \; -i \frac{N_c}{240\pi^2} \int \; d^5x \; \varepsilon^{\mu\nu\alpha\beta\gamma} \; \operatorname{Tr}(L_\mu L_\nu L_\alpha L_\beta L_\gamma) \ , \tag{3}$$

which distinguishes between states with positive and negative strangeness.

We proceed by introducing the Callan–Klebanov (CK) ansatz for the chiral field [9]

$$U = \sqrt{U_\pi} U_K \sqrt{U_\pi} \ . \tag{4}$$

In this ansatz, U_K is the field that carries the strangeness. Its form is

$$U_K \; = \; \exp\left[i \frac{2\sqrt{2}}{\mathrm{F}_K} \begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix} \right] \ , \tag{5}$$

where K is the usual kaon isodoublet

$$K \; = \; \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \ . \tag{6}$$

The other component, U_π, is the soliton background field. It is a direct extension to $SU(3)$ of the $SU(2)$ field u_π, i.e.,

$$U_\pi = \begin{pmatrix} u_\pi & 0 \\ 0 & 1 \end{pmatrix} . \tag{7}$$

Replacing the ansatz Eq.(4) in the effective action Eq.(1) and expanding up to second order in the kaon fields we obtain the following lagrangian density for the kaon–soliton system

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{SU(2)} + (D_\mu K)^\dagger (D^\mu K) - K^\dagger a_\mu a^\mu K \\ &- \frac{2}{e^2 {\mathrm{F}_K}^2} \left\{ K^\dagger K \ \mathrm{Tr} \left([a_\mu, a^\nu]^2 \right) - (D_\mu K)^\dagger (D^\mu K) \mathrm{Tr}(a_\nu a^\nu) \right. \\ &+ \left. (D_\mu K)^\dagger (D_\nu K) \mathrm{Tr}(a^\mu a^\nu) - 3 (D_\mu K)^\dagger [a^\mu, a^\nu] (D_\nu K) \right\} \\ &- i \frac{N_c}{{\mathrm{F}_K}^2} B^\mu \left[K^\dagger D_\mu K - (D_\mu K)^\dagger K \right] \\ &- K^\dagger K \left[{m_K}^2 - \frac{1}{2} \frac{{\mathrm{F}_\pi}^2}{{\mathrm{F}_K}^2} {m_\pi}^2 (1 - \cos F) \right] \end{aligned} \tag{8}$$

where $\mathcal{L}_{SU(2)}$ is the effective pion lagrangian whose explicit expression is given in the Appendix A and

$$\begin{aligned} D_\mu &= \partial_\mu + v_\mu , \\ \begin{pmatrix} v_\mu \\ a_\mu \end{pmatrix} &= \frac{1}{2} (\sqrt{u_\pi}^\dagger \partial_\mu \sqrt{u_\pi} \pm \sqrt{u_\pi} \partial_\mu \sqrt{u_\pi}^\dagger) . \end{aligned} \tag{9}$$

B^μ is the baryon number current of the $SU(2)$ configuration given by

$$B^\mu = \frac{1}{24\pi^2} \varepsilon^{\mu\alpha\beta\gamma} \ \mathrm{Tr}(l_\alpha l_\beta l_\gamma) , \tag{10}$$

where $l_\nu = u_\pi^\dagger \partial_\nu u_\pi$.

In order to obtain the soliton background configuration we introduce the axially symmetric ansatz

$$u_\pi = \exp \left[i \vec{\tau} \cdot \hat{\pi}_n \ F \right] \tag{11}$$

with

$$\hat{\pi}_n = \sin\Theta \cos n\phi \ \hat{\imath} + \sin\Theta \sin n\phi \ \hat{\jmath} + \cos\Theta \ \hat{k} \tag{12}$$

In Ref.[12] it was assumed that $F = F(r)$ and $\Theta = \theta$, where (r, θ, ϕ) are the usual spherical coordinates. As already mentioned, such an ansatz, which predicts

$$R \equiv M_{sol} \ (B = 2) \ / \ M_{sol} \ (\ B = 1 \ ) = 2.14 , \tag{13}$$

leads to an unstable soliton configuration. The lowest energy diskyrmion configuration was numerically found in Ref.[15]. A very good variational approximation to such a solution

was proposed in Ref.[14]. In this case, F is a function of r only, but the variational function

$$\Theta = \theta + \sum_{k=1}^{m} g_k \ \sin(2k\theta) \tag{14}$$

is used for Θ. The coefficients g_k are determined by minimizing the soliton energy in the corresponding baryon sector. Using this ansatz one finds $R = 1.94$, which compares very well with the lowest energy solution value $R_{min} = 1.92$. As already mentioned, other computed quantities like rotational energies, baryon radius, etc. are numerically also very close to those of Ref.[15]. Since the use of the variational ansatz leads to a considerable simplification of the calculation, we will use it to describe the soliton background. The expressions corresponding to the $SU(2)$ sector of the model have been obtained in Ref.[14]. For completeness, they are summarized in Appendix A.

Of course, for $n = 1$ the minimum soliton energy is obtained for $g_k = 0$. Therefore, in this case the background soliton is still symmetric under combined spatial and isospin rotations $\vec{\Lambda} = \vec{l} + \vec{I}$ and the kaon field can be expanded in terms of spinor spherical harmonics,

$$K(\vec{r},t) \ = \ k(r,t) \ \mathcal{Y}_{\Lambda l \Lambda_3}(\hat{r}) \ . \tag{15}$$

However, for $n \neq 1$ the background field is no longer invariant under Λ–rotations. In this case we use the consistent ansatz [12]

$$K(\vec{r},t) \ = \ k(r,t) \ \vec{\tau} \cdot \hat{\pi}_n(\vec{r}) \ \chi \ , \tag{16}$$

where χ is a two–component spinor. Note that, for $n = 1$, this ansatz reduces to Eq.(15) for the particular case $\Lambda = 1/2$, $l = 1$. These are precisely the quantum numbers of lowest kaon bound state when $n = 1$.

Using the ansätze given above the explicit form of the kaon-soliton effective lagrangian is

$$L \ = \ \int \ dr \ r^2 \left\{ f(r) \ \dot{k}^\dagger \dot{k} - h(r) \ k^{\dagger\prime} k' + i\lambda(r)(\dot{k}^\dagger k - k^\dagger \dot{k}) - k^\dagger k ({m_K}^2 + V_{eff}) \right\} \ , \tag{17}$$

where

$$f(r) \ = \ 1 + \frac{1}{e^2 {\mathrm{F}_K}^2} \left(F'^2 + \alpha_1 \frac{\sin^2 F}{r^2} \right) \ , \tag{18}$$

and

$$h(r) \ = \ 1 + \frac{\alpha_1}{e^2 {\mathrm{F}_K}^2} \frac{\sin^2 F}{r^2} \ . \tag{19}$$

The term linear in time derivatives, whose coefficient is

$$\lambda(r) \ = \ -\frac{\alpha_3 \ N_c}{2\pi^2 {\mathrm{F}_K}^2} \ F' \ \frac{\sin^2 F}{r^2} \ , \tag{20}$$

is due to the Wess–Zumino action, and

$$
\begin{aligned}
V_{eff} &= \left[\frac{\alpha_1}{e^2 {\rm F}_K{}^2 r^2}\left(\cos^4 F/2 - 2\ \sin^2 F\right) - \frac{1}{4}\right]\ F'^2 \\
&- \frac{1}{r^2}\left(\sin^2 F - 4\ \cos^4 F/2\right)\left(\frac{2\alpha_2}{e^2 {\rm F}_K{}^2}\frac{\sin^2 F}{r^2} + \frac{\alpha_1}{4}\right) \\
&- 3\frac{\alpha_1}{e^2 {\rm F}_K{}^2 r^2}\frac{d}{dr}\left[F' \sin F\ \cos^2 F/2)\right] - \frac{1}{2}\frac{{\rm F}_\pi{}^2}{{\rm F}_K{}^2} m_\pi{}^2 (1-\cos F)\ . \qquad (21)
\end{aligned}
$$

Here, α_i are

$$
\begin{aligned}
\alpha_1 &= \frac{1}{2}\int_0^\pi d\theta\ \sin\theta\left(\Theta'^2 + n^2\ \frac{\sin^2\Theta}{\sin^2\theta}\right)\ , \\
\alpha_2 &= \frac{n^2}{2}\int_0^\pi d\theta\left(\Theta'^2\ \frac{\sin^2\Theta}{\sin\theta}\right)\ , \\
\alpha_3 &= \frac{n}{2}\int_0^\pi d\theta\ \sin\Theta\ \Theta'\ . \qquad (22)
\end{aligned}
$$

The diagonalization of the hamiltonian obtained from the effective lagrangian Eq.(17) leads to the kaon eigenvalue equation

$$
\left[-\frac{1}{r^2}\partial_r\left(r^2 h \partial_r\right) + m_K^2 + V_{eff} - f\varepsilon^2 - 2\ \lambda\ \varepsilon\right] k(r) = 0\ . \qquad (23)
$$

To obtain the hyperfine corrections to the dibaryons masses we proceed with the semiclassical collective coordinates quantization method, where the isospin and spatial rotations are treated as the zero modes [20]. Then, we introduce the time–dependent spatial rotations R and the isospin rotations A such that

$$
\begin{aligned}
u_\pi &\rightarrow R\ A\ u_\pi\ A^{-1} \qquad (24) \\
K &\rightarrow R\ A\ K\ . \qquad (25)
\end{aligned}
$$

The angular velocities in respect to the body fixed frame are given by

$$
\begin{aligned}
\left(R^{-1}\dot{R}\right)_{ab} &= \varepsilon_{abc}\Omega_c \qquad (26) \\
A^{-1}\dot{A} &= \frac{i}{2}\ \vec{\tau}\cdot\vec{\omega}\ . \qquad (27)
\end{aligned}
$$

Using a_1 and a_2 as coefficients of the up and down spinor χ in Eq.(16) the substitution of Eqs.(24,25) in the full Lagrangian Eq.(8) yields

$$
\begin{aligned}
L &= -M_{sol} + L_K - \vec{T}\cdot\vec{\omega} + (T_1\omega_1 + T_2\omega_2)s_2 + T_3\omega_3 s_1 - (T_1\Omega_1 + T_2\Omega_2)t_2 - T_3\Omega_3 t_1 \\
&+ \frac{1}{2}\mathcal{I}_1(\Omega_1^2+\Omega_2^2) + \frac{1}{2}\mathcal{I}_2(\omega_1^2+\omega_2^2) - \mathcal{I}_4\delta_{n,1}(\Omega_1\omega_1+\Omega_2\omega_2) + \frac{1}{2}\mathcal{I}_3(n\Omega_3-\omega_3)^2\ , (28)
\end{aligned}
$$

where [1]

$$s_1 = 3\left\{2\alpha_4\ d_1\ +\ [\alpha_5 - \alpha_1\alpha_4)]\ d_2\right\}\ , \tag{29}$$

$$s_2 = 3\left\{(1-\alpha_4)\ d_1\ +\ \frac{1}{2}[\alpha_1(1+\alpha_4) - \alpha_5)]\ d_2\right\}\ , \tag{30}$$

$$t_1 = n\ s_2\ , \tag{31}$$

$$t_2 = 2(d_1\ +\ d_2)\ \delta_{n,1}\ , \tag{32}$$

with d_1 and d_2 given by

$$d_1 = 2\varepsilon_n \int_0^\infty dr\ k^* k\ \left[\frac{2}{3} r^2 f \cos^2 F/2\ -\ \frac{1}{e^2 {\mathrm{F}_K}^2}\frac{d}{dr}(r^2\ F' \sin F)\right] \tag{33}$$

$$d_2 = \frac{2\varepsilon_n}{e^2 {\mathrm{F}_K}^2} \int_0^\infty dr\ k^* k\ \frac{4}{3}\cos^2 F/2\ \sin^2 F\ , \tag{34}$$

and the angular integrals α_4 and α_5 by

$$\alpha_4 = \frac{1}{4}\int_0^\pi d\theta\ \sin\theta\ \sin^2\Theta\ , \tag{35}$$

$$\alpha_5 = \int_0^\pi d\theta\ \sin\theta\ \sin^2\Theta\ {\Theta'}^2\ . \tag{36}$$

In Eq.(28) T^l is defined as $T^l = a_i^* \tau_{ij}^l a_j$ and $\mathcal{I}_i$ are the moments of inertia of $SU(2)$ sector whose explicit expressions are given in Appendix A. The spin and isospin components J_i^{bf} and I_i^{bf} respectively, are calculated via

$$J_i^{bf} = \frac{\partial L}{\partial \Omega_i}\ , \qquad I_i^{bf} = \frac{\partial L}{\partial \omega_i}\ . \tag{37}$$

Unlike the $n = 1$ case where the hedgehog symmetry of the skyrmion enforces the constraint $\vec{J} + \vec{I} = \vec{T}$, for $n \neq 1$ only the constraint

$$J_3^{bf} = -n(I_3^{bf}\ +\ T_3) \tag{38}$$

due to axial symmetry is left.

The quantization of the rotational hamiltonian leads to

$$\begin{aligned} H_{rot} &= \frac{1}{2\mathcal{I}_1}(\vec{J}^2 - (J_3^{bf})^2)\ +\ \frac{1}{2\mathcal{I}_2}(\vec{I}^2 - (I_3^{bf})^2)\ +\ \frac{c_2^2}{2\mathcal{I}_2}(\vec{T}^2 - T_3^2) \\ &+ \frac{1}{2\mathcal{I}_3}(I_3^{bf} + c_1 T_3)^2\ +\ \frac{c_2}{2\mathcal{I}_2}(I_+^{bf} T_- + I_-^{bf} T_+)\ , \end{aligned} \tag{39}$$

with the hyperfine constants c_1 and c_2 given by

$$c_1 = 1\ -\ s_1 \tag{40}$$

$$c_2 = 1\ -\ s_2\ . \tag{41}$$

Given this form of the rotational hamiltonian we should find the corresponding eigenfunctions that satisfy the constraints imposed by the symmetries of the system. This is done in the following section.

[1] In what follows we assume that $\Theta = \theta$ for $n = 1$.

3 Dibaryon wavefunctions

In general the dibaryon wave functions are combinations of product states of the rotation matrices $D^I_{I_3,I_3^{bf}}(\omega)$ for isospin, $D^J_{J_3,J_3^{bf}}(\Omega)$ for the angular momentum and the kaon eigenstates $k_{T_3}(\vec{r},t)$. Here I_3 and J_3 are respectively the isospin and angular momentum projection on the lab. frame, I_3^{bf} and J_3^{bf} on the body fixed frame and T_3 the projection of the kaon "spin" on the body fixed frame. Each of these product states have the form

$$D^J_{J_3,J_3^{bf}}(\Omega)\ D^I_{I_3,I_3^{bf}}(\omega)\ k_{T_3}(\vec{r},t) \tag{42}$$

where

$$J_3^{bf} = -2(I_3^{bf} + T_3^{bf}). \tag{43}$$

First we note that in these product states, for a given value of S, not all the values of isospin are allowed. This can be shown using the same method used in Ref.[16] for $B = 1$. Details are given in Appendix B. One obtains that for states in the minimal representations the allowed values of isospin are given by

$$I = \frac{p}{2} \tag{44}$$

where $p \leq 6 + S$ should be odd if S is odd or even otherwise. This relation together with the axial symmetry constraint Eq.(43) imply that J_3^{bf} is always even.

To determine for each set of quantum numbers the correct linear combination of product states we should take into account all the symmetries of the problem. As discussed in Ref.[14] in addition to axial symmetry, the ansatz Eqs.(12,14) has, for $n = 2$, the following reflection symmetries

$$\hat{\pi}_2(-x,y,z) = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \hat{\pi}_2(x,y,z) \tag{45}$$

$$\hat{\pi}_2(x,-y,z) = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \hat{\pi}_2(x,y,z) \tag{46}$$

$$\hat{\pi}_2(x,y,-z) = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \hat{\pi}_2(x,y,z) \tag{47}$$

Denoting spatial rotations through an angle ϕ about the i-axis by $\mathcal{R}^J_i(\phi)$ and isorotations through an angle θ about the j-axis by $\mathcal{R}^I_j(\theta)$ the symmetry transformations are also generated by

$$\mathcal{R}^J_1(\pi)\ \mathcal{R}^I_1(\pi) \tag{48}$$

$$\mathcal{R}^J_2(\pi)\ \mathcal{R}^I_1(\pi) \tag{49}$$

$$\mathcal{P}\ \mathcal{R}^I_3(\pi) \tag{50}$$

The operator $\mathcal{P}$ denotes the parity operator which is defined as a space inversion and a sign change of all components of the pion field.

The importance of the first two of these symmetry transformations on the construction of the dibaryon wavefunctions was first noticed in Ref.[17]. In fact, as it discussed in detail in Sec.4-2c of Ref.[18], for system with axial symmetry, the existence of an additional symmetry with respect to a π-rotation about an axis perpendicular to the symmetry axis implies that this π-rotation is part of the intrinsic degrees of freedom, and is therefore not to be included in the rotational degrees of freedom. We can express this constraint by requiring that the operator $\mathcal{R}_{ext}$, which performs a rotation $\mathcal{R}$ by acting on the collective orientation angles (external variables), is identical to the operator $\mathcal{R}_{int}$, which performs the same rotation by acting on the intrinsic variables,

$$\mathcal{R}_{ext} = \mathcal{R}_{int} \ . \tag{51}$$

Since in our case $\mathcal{R}$ can be either given by Eq.(48) or Eq.(49) we will have, in principle, two independent constraints. We will see, however, that when applied to the product states both symmetry operations produce the same result and therefore we are left with one single constraint.

Then, in order to satisfy the constraints imposed by the symmetries of the ansatz we have to choose the following linear combination of product states:

$$N \left[1 + (\mathcal{R}_{ext})^{-1}\mathcal{R}_{int}\right] D^J_{J_3,J_3^{bf}}(\Omega)\ D^I_{I_3,I_3^{bf}}(\omega)\ k_{T_3}(\vec{r},t) \ , \tag{52}$$

where N is a normalization factor. In writing Eq.(52) we have use the fact that acting on product states $(\mathcal{R}_{ext})^2 = (\mathcal{R}_{int})^2$. As we will see below this is satisfied by our product states.

In order to have the explicit form of the dibaryon wave function we should apply the symmetry operators on the product wave function. For the collective operators we get

$$\left[\mathcal{R}^J_1(\pi)\mathcal{R}^I_1(\pi)\right]^{-1} D^J_{J_3,J_3^{bf}}(\Omega) D^I_{I_3,I_3^{bf}}(\omega) \ = \ (-)^{I+J} D^J_{J_3,-J_3^{bf}}(\Omega) D^I_{I_3,-I_3^{bf}}(\omega) \ , \tag{53}$$

$$\left[\mathcal{R}^J_2(\pi)\mathcal{R}^I_1(\pi)\right]^{-1} D^J_{J_3,J_3^{bf}}(\Omega) D^I_{I_3,I_3^{bf}}(\omega) \ = \ (-)^{I+J-J_3^{bf}} D^J_{J_3,-J_3^{bf}}(\Omega) D^I_{I_3,-I_3^{bf}}(\omega) \ . \tag{54}$$

To determine the effect of the symmetry transformations on the intrinsic wave function we have to notice that apart from the contribution of the kaon field we have to include the contribution of the soliton itself. The latter contribution has been calculated in Refs.[17, 19]. Using the fermionic nature of the $B = 1$ soliton configuration they found[2]

$$\mathcal{R}^J_1(\pi)\mathcal{R}^I_1(\pi)\ \psi_{sol} \ = \ -\ \psi_{sol} \ , \tag{55}$$

$$\mathcal{R}^J_2(\pi)\mathcal{R}^I_1(\pi)\ \psi_{sol} \ = \ -\ \psi_{sol} \ . \tag{56}$$

[2]Note that when one extends the light flavour group from $SU(2)$ to $SU(3)^f$ as done in Ref.[16] the minus phase is obtained from the Wess-Zumino term.

Finally, we have to calculate the effect of the symmetry transformations on the kaon field. Performing the symmetry operations on one kaon states we find

$$\begin{aligned}
\mathcal{R}^J_1(\pi)\mathcal{R}^I_1(\pi)\ k_{T_3}(S=\pm 1) &= \mp i\ k_{-T_3}(S=\pm 1)\ , & (57)\\
\mathcal{R}^J_2(\pi)\mathcal{R}^I_1(\pi)\ k_{T_3}(S=\pm 1) &= \mp i\ k_{-T_3}(S=\pm 1)\ . & (58)
\end{aligned}$$

In deriving Eqs.(57,58), we have use the form Eq.(16) for $S=+1$ kaons and its charge conjugate for $S=-1$ kaons. Therefore, for a state with arbitrary value of strangeness S we obtain

$$\begin{aligned}
\mathcal{R}^J_1(\pi)\mathcal{R}^I_1(\pi)\ k_{T_3} &= (-)^{-S/2}k_{-T_3}\ , & (59)\\
\mathcal{R}^J_1(\pi)\mathcal{R}^I_1(\pi)\ k_{T_3} &= (-)^{-S/2}k_{-T_3}\ . & (60)
\end{aligned}$$

As we see, the only difference between the action of the symmetry transformation Eq.(48) and that of Eq.(49) is the presence of an extra phase $(-)^{J_3^{bf}}$ in Eq.(54). Since we have shown that in our case J_3^{bf} is always even, we see that, as stated above, both symmetry operations produce that same effect when acting on our product states. Moreover, using the constraints Eqs.(43,44) together with the effect of the symmetry operations on the intrinsic and collective wave functions Eqs.(53-60), it is easy to show that $\mathcal{R}^2_{ext}=\mathcal{R}^2_{int}$ when applied to the product states.

Therefore, the dibaryon wave functions that satisfy all the constraints imposed by the symmetries of the system have the structure

$$\begin{aligned}
|II_3, JJ_3, S> &= N\Big(D^J_{J_3,-2K}(\Omega)D^I_{I_3,K-T_3}(\omega)k_{T_3}(\vec{r},t)- \\
&\qquad (-)^{I+J-S/2}D^J_{J_3,2K}(\Omega)D^I_{I_3,-K+T_3}(\omega)k_{-T_3}(\vec{r},t)\Big)\ , \qquad (61)
\end{aligned}$$

where $K=I_3^{bf}+T_3$. The normalization constant N can be easily calculated. We obtain

$$N = \frac{1}{\sqrt{2(1+\delta_{I_3^{bf},0}\delta_{T_3,0})}}\frac{\sqrt{(2J+1)(2I+1)}}{8\pi^2}\ . \qquad (62)$$

To determine the parity of these wavefunctions we have to make use of the symmetry operation given in Eq.(50). In fact, as discussed in Sec.4-2f of Ref.[18] the existence of such kind of symmetries implies that the parity operator $\mathcal{P}$ can be written as

$$\mathcal{P}=\mathcal{S}\ \mathcal{R}^{-1}\ , \qquad (63)$$

where $\mathcal{S}$ ($=\mathcal{P}\mathcal{R}^I_3(\pi)$ in our case) acts on the intrinsic coordinates, while $\mathcal{R}^{-1}$ ($=[\mathcal{R}^I_3(\pi)]^{-1}$ in our case) acts on the collective variables. Using the fact that K is always integer and that

$$\begin{aligned}
\mathcal{S}\ \psi_{sol} &= \psi_{sol} & (64)\\
\mathcal{S}\ k_{\pm T_3} &= (-)^{\pm T_3}\ k_{\pm T_3} & (65)\\
\mathcal{R}^I_3(\pi)\ D^J_{J_3,\mp 2K}(\Omega)D^I_{I_3,\pm(K-T_3)}(\omega) &= (-)^{\pm(K-T_3)}\ D^J_{J_3,\mp 2K}(\Omega)D^I_{I_3,\pm(K-T_3)}(\omega) & (66)
\end{aligned}$$

we find that the parity of the wavefunction Eq.(61) is given by

$$\pi = (-)^K. \tag{67}$$

From the explicit form of the wavefunction we note that for $K = 0$, $T_3 = 0$ only the "Fermi–allowed" combinations (satisfying the constraint $(-)^{I+J-S/2} = -1$) survive, whereas the "Fermi–forbidden" ones have zero norm. For the particular case of the two-nucleon system ($S = 0$), we have the constraint $(-)^{I+J} = -1$ which is nothing but the generalized form of the Pauli principle. It is interesting to note that in our case this principle realizes in a more conventional way than in Ref.[19] (see Eqs.(5.7a-b) of that reference). This is due to the more convenient choice of spatial coordinates we have used in the present work. In general, using the constraint Eq.(44) together with the explicit form of the dibaryon eigenstates Eq.(61), one readily finds that all the spurious states obtained in Refs.[12, 13, 14] are removed from the spectrum. The quantum numbers of the allowed states with $J \leq 2$ are shown in Table 1.

An important remark is that the wavefunction obtained above is not an eigenfunction of the rotational hamiltonian H_{rot}. Since the Coriolis term (last term in Eq.(39)) does not commute with T_3, the eigenfunctions of the H_{rot} will be combinations of those given in Eq.(61) with the same II_3, JJ_3, $(J_3^{bf})^2$ quantum numbers, but different values of T_3^2.

Using Eqs.(39,61) one can finally obtain the dibaryon mass formula. For ground state dibaryons we get

$$M = M_{sol} + |S|\, \varepsilon + M_{rot}(S) \tag{68}$$

where S is the strangeness of the state and $M_{rot}(S)$ is the rotational contribution. Since for $S = 0$ and $S = -1$ only one value of T_3^2 is allowed ($T_3^2 = 0$ for $S = 0$ and $T_3^2 = 1/4$ for $S = -1$), in these cases M_{rot} is simply given by the mean value of H_{rot} in the corresponding dibaryon state. For $S = 0$ we get

$$M_{rot}^{S=0} = \frac{1}{2\mathcal{I}_1}\left[J(J+1) - (J_3^{bf})^2\right] + \frac{1}{2\mathcal{I}_2}\left[I(I+1) - (I_3^{bf})^2\right] + \frac{1}{2\mathcal{I}_3}(I_3^{bf})^2\,, \tag{69}$$

i.e., the same as [13].

For $S = -1$, the rotational contribution is given by

$$\begin{aligned} M_{rot}^{S=-1} &= \frac{1}{2\mathcal{I}_1}\left[J(J+1) - (J_3^{bf})^2\right] + \frac{1}{2\mathcal{I}_3}\left\{(1-c_1)\left[(I_3^{bf})^2 - \frac{c_1}{4}\right] + \frac{c_1 (J_3^{bf})^2}{4}\right\} \\ &+ \frac{1}{2\mathcal{I}_2}\left[I(I+1) - (I_3^{bf})^2 + c_2\left(\frac{c_2}{2} - (-)^{I+J+1/2}\delta_{J_3,0}\sqrt{I(I+1)+1/4}\right)\right] \end{aligned} \tag{70}$$

For $S = -2$, two values of T_3^2 ($= 0$, 1) are in general allowed [3]. Therefore, M_{rot} is given by the eigenvalues of

$$M_{rot}^{S=-2} = \frac{1}{2\mathcal{I}_1}\left[J(J+1) - (J_3^{bf})^2\right] + \frac{1}{2\mathcal{I}_2}\left[I(I+1) + c_2^2\right] + \frac{c_1 K^2}{2\mathcal{I}_3}$$

[3] Whether *both* values are allowed for a given state depends on the values of the other quantum numbers.

$$+\begin{pmatrix} \frac{c_2^2-K^2}{2\mathcal{I}_2}+\frac{1-c_1}{2\mathcal{I}_3}K^2 & \frac{c_2}{\mathcal{I}_2\sqrt{2}}\sqrt{1+\delta_{K,0}}\sqrt{(I\mp K)(I\pm M)} \\ \frac{c_2}{\mathcal{I}_2\sqrt{2}}\sqrt{1+\delta_{K,0}}\sqrt{(I\mp K)(I\pm M)} & \frac{1-c_1}{2\mathcal{I}_3}\left(M^2-c_1\right)-\frac{M^2}{2\mathcal{I}_2} \end{pmatrix} \qquad (71)$$

with $M = K \pm 1$. In the construction of 2 x 2 matrix we have used the basis given in Eq.(61) ordered according to increasing T_3^2.

Note that although in some particular cases the expressions for the rotational corrections to $S = -1$ and $S - 2$ dibaryons agree with those given in Ref.[12], in general they differ. This is due to the fact that in general the eigenfunctions used in that reference do not satisfy the constraints imposed by the symmetry transformations Eqs.(48,49).

4 Results and discussion

In our numerical calculations we will consider two sets of values for the parameters in the effective action. In one case (SET A), we consider the chiral limit in the $SU(2)$ sector, $m_\pi = 0$ and fit F_π and e to reproduce the empirical N and Δ masses. This corresponds to the result of Ref.[20],

$$F_\pi = 129\ MeV, \qquad e = 5.45\ . \qquad (72)$$

The second set of parameters (SET B) is obtained for $m_\pi = 138\ MeV$. It corresponds to the result of Ref.[21]

$$F_\pi = 108\ MeV, \qquad e = 4.84\ . \qquad (73)$$

In both cases we take the kaon mass and the ratio F_K/F_π to their empirical values $m_K = 495\ MeV$, $F_K/F_\pi = 1.22$. When comparing our results with those of Ref.[12], it should be noticed that in that reference the ratio of decay constants was taken to be 1. As mentioned above this leads to an important overbinding in the $B = 1$ sector. The spectrum of non-strange and strange baryons obtained for our two parameters sets can be found for example in Ref.[10] and it will be not repeated here.

As shown in Ref.[14] when SET B is used and only g_1 is taken as variational parameter, the minimum in the $B = 2$ soliton mass is found for

$$g_1 = -0.339 \qquad g_{i\neq 1} = 0\ . \qquad (74)$$

The inclusion of g_2 as a second variational parameter does not lead to any significant improvement in the energy minimum. In addition we have also checked that for SET A the minimum appears almost at the same value of g_1. For this reason we will use the set of g_i given in (74) in all our calculations.

In the Table 2, we present the calculated values of all the parameters appearing in the dibaryon mass formulae for both the massless and the massive pion cases. Non-strange sector parameters corresponding to SET B have already been given in Ref.[14]. They are repeated here for completeness. The values of ε indicate that kaons are less bound to the soliton than in the $B = 1$ case. Using the $B = 1$ kaon eigenenergies given in Ref.[11], we get

$$\Delta\varepsilon \equiv \varepsilon(B = 2) - \varepsilon(B = 1) = 16\ MeV \qquad (75)$$

for both, SET A and SET B. A similar result was found in Ref.[12], although in that case the value of $\Delta\varepsilon$ was somewhat smaller, $\Delta\varepsilon \simeq 10\ MeV$. It should be noticed that when the empirical value of the meson decay constant ratio is used in order to eliminate the large kaon overbinding found in Ref.[12], $\Delta\varepsilon$ becomes smaller. In fact, if we set all $g_i = 0$ and use SET A as done in Ref.[12] but keep $F_K/F_\pi = 1.22$, we find $\Delta\varepsilon = 0$. On the other hand, the use of the improved axially symmetric ansatz increases the value of $\Delta\varepsilon$, the effect being larger for $F_K/F_\pi = 1.22$ (=16 MeV) than for $F_K/F_\pi = 1$ (= 8 MeV). Similar comments hold for SET B.

The calculated rotational corrections M_{rot} to the dibaryon masses are shown in Table 3. We list only the allowed states with $M_{rot} \leq 250\ MeV$. For comparison, we also give the results of Ref.[12] and the sum of the rotational contribution to the lowest baryon-baryon state in each particular channel. We observe that our rotational energies are somewhat dependent on the choice of parameters, being smaller for SET B. For SET A they are in general very similar to those reported in Ref.[12]. An important exception is the lowest $S = -2$ state for which we predict roughly half of the value given there. The reason for this difference is mainly the smaller value of c_2 obtained in our model. This is of some importance since in our calculation the rotational energy of this state lies below the corresponding threshold for both sets of parameters in contrast with the situation in Ref.[12]. Another interesting $S = -2$ state to be discussed is the $(1, 1^+)$ state. This is lowest state for which the off-diagonal terms in Eq.(71) do not vanish. Therefore, in order to calculate the corresponding rotational energy the 2 x 2 matrix has been diagonalized. It should be noticed than in doing so one goes beyond the $\mathcal{O}(N_c^{-1})$ in the $1/N_c$ expansion (where N_c is the number of colors)[4]. Strictly speaking this is not consistent with the fact that other contributions to the same order have been systematically neglected. However, if we just consider the corresponding diagonal term in Eq.(71) as the rotational correction to such an state one obtains $M_{rot}(1, 1^+) = 154\ MeV$ (SET A). This rather large rotational correction is in disagreement with different calculations (see i.e. Ref.[8]) that predict the existence of a low-lying state with these quantum numbers.

Now we will focus our attention on the problem of the H stability. The different contributions to the corresponding binding energy are given in Table 4. As we see, for both sets of parameters, the H is bound in our model. Since the attraction found at the level of hyperfine (rotational) interactions is not enough to compensate the fact that kaons are less bound to the diskyrmion, the main source of attraction turns out to be the rather low value of the $B = 2$ soliton mass obtained with the improved axially symmetric ansatz. A similar situation occurs in soliton models calculation based on the $SU(3)$ collective coordinates approach. At this point it is important to recall that although the lowest energy state in the $S = 0$ channel has the quantum numbers of the deuteron its identification with this particle it is not completely clear. In particular, a large binding ($\simeq 130 MeV$) is predicted for this state in the present model [14]. Since a considerable part of this overbinding comes from the $\mathcal{O}(N_c)$ contribution to the dibaryon mass this might be an indication that the static axial symmetric torus does not provide the correct

[4]For a similar situation in a slightly different context see Ref.[11]

description of the $B = 2$ baryons. In using this configuration only the potential energy aspect of the dibaryon state is addressed, but all of the relative kinetic energy contribution has been neglected. In fact, it has been recently shown in Ref.[22] where a bound two-skyrmion configuration was numerically studied on a discrete mesh, the torus state was formed only at the closest encounter of the two skyrmions, whereas most of the time the two-skyrmion system consisted out of two well separated baryons as expected for a "well-respecting" deuteron. Another effect that has been ignored in our calculation (and which is possible related with the lack of kinetic corrections mentioned above) is the Casimir effect due to zero point vibrations. A recent estimation of this effect using the $SO(3)$ ansatz [23] indicates that it tends to shift the mass of the $B = 2$ configuration relative to two $B = 1$ hedgehogs upwards. If we assume that this is the main source of repulsion that brings the deuteron mass to its physical value, then such a repulsion would be enough to unbind the H-dibaryon.

5 Conclusions

In this article we have studied the structure of strange dibaryons in the context of the bound state soliton model. In this approach, such dibaryons are assumed to be bound states of kaons and a $B = 2$ topological soliton. To describe the diskyrmion configuration we have used an axially symmetric ansatz where the dependence on the azimuthal angle θ is found through a variational method. Such an ansatz provides a very good approximation to the numerically found lowest energy solution which also has axial symmetry. We have shown that once the constrains imposed by the symmetries of the torus background configuration are satisfied all spurious states are eliminated from the spectrum. In particular, we obtained a generalized form of the Pauli principle for the lowest lying $B = 2$ states with even values of strangeness. In the $S = 0$ sector this implies that the lowest allowed state has the quantum numbers of the deuteron, while for the $S = -2$ sector the lowest state has the quantum numbers of the H particle. This is in agreement with the predictions of the quark based models.

We have found that although kaons are less bound to the diskyrmion configuration than to a single soliton, the H-dibaryon is barely bound within our approximations. This binding is mainly due to the rather small diskyrmion mass with respect to two individual skyrmions. However, we know from the case of the deuteron (which is strongly overbound in the present model[14]) that the static torus configuration tends to underestimate the $B = 2$ soliton mass. Moreover, numerical studies [22] showed that the torus configuration is formed only at the closest encounter of two skyrmions. In this sense it would be interesting to see whether the dynamical departures from the lowest energy solution can be parametrized in terms of some collective coordinate. Zero-point fluctuations of this coordinate could then provide a mechanism to increase the $B = 2$ soliton mass without affecting the $B = 1$ skyrmion mass. Another effect that is expected to decrease the binding energies (and which is probably very much related with the previous one) is the Casimir effect which leads to contributions of $\mathcal{O}(N_c^0)$ [23]. Since these effects are expected

to give similar contributions to all the dibaryon states independently of their strangeness, it is clear that if they are strong enough to push the deuteron mass to its empirical value they are very likely to unbind the H.

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Appendix A: The $SU(2)$ Sector

In this appendix, we give, for completeness the expressions for the $SU(2)$ sector of our model. Most of them have been already published elsewhere [14].

The lagrangian density $\mathcal{L}_{SU(2)}$ in Eq.(8) is

$$\begin{aligned}\mathcal{L}_{SU(2)} &= \frac{\mathrm{F}_\pi{}^2}{16}\mathrm{Tr}(\partial_\mu u_\pi{}^\dagger \partial^\mu u_\pi) + \frac{1}{32e^2}\mathrm{Tr}\left([\partial_\mu u_\pi u_\pi{}^\dagger, \partial_\nu u_\pi u_\pi{}^\dagger][\partial^\mu u_\pi u_\pi{}^\dagger, \partial^\nu u_\pi u_\pi{}^\dagger]\right) \\ &-\frac{1}{4}\mathrm{F}_\pi{}^2 m_\pi{}^2(1-\cos F) \ . \end{aligned} \tag{A.1}$$

Replacing the modified ansatz given in Eq.(11) the classical mass turns out to be a functional of F, Θ and Φ which explicit expression is

$$\begin{aligned} M[F,\Theta,\Phi] &= \int d^3\vec{r}\ \left\{ \frac{\mathrm{F}_\pi{}^2}{8}\left[F'^2 + \left(\Theta'^2 + \frac{\sin^2\Theta}{\sin^2\theta}\ \Phi'^2\right)\frac{\sin^2 F}{r^2}\right]\right. \\ &+ \frac{1}{2e^2}\frac{\sin^2 F}{r^2}\left[\left(\Theta'^2 + \frac{\sin^2\Theta}{\sin^2\theta}\ \Phi'^2\right)\ F'^2\right. \\ &+ \left.\left.\frac{\sin^2\Theta}{\sin^2\theta}\ \Theta'^2\ \Phi'^2\ \frac{\sin^2 F}{r^2}\right] + \frac{m_\pi{}^2\mathrm{F}_\pi{}^2}{4}(1-\cos F)\right\}\ , \end{aligned} \tag{A.2}$$

with

$$F' = \frac{dF}{dr}\ , \qquad \Theta' = \frac{d\Theta}{d\theta} \qquad \text{and} \qquad \Phi' = \frac{d\Phi}{d\phi}\ . \tag{A.3}$$

The minimization of Eq.(A.2) leads to an Euler–Lagrange for each of the functions F, Θ and Φ. The first, and more simple, is

$$\Phi'' = 0\ . \tag{A.4}$$

Due to single–valueness of the chiral field we must have $\Phi = n\varphi$, with n the baryon number. Using this result for Φ, the equation for Θ is

$$\begin{aligned} &\left(C_1\sin\theta + C_2\frac{n^2}{\sin\theta}\sin^2\Theta\right)\Theta'' + \left(C_1 - C_2 n^2\frac{\sin^2\Theta}{\sin^2\theta}\right)\cos\theta\ \Theta' \\ &- \left(C_1 - C_2\Theta'^2\right)n^2\frac{\sin\Theta}{\sin\theta}\cos\Theta = 0\ , \end{aligned} \tag{A.5}$$

with

$$C_1 = 2\pi\int_0^\infty dr\ \sin^2 F\ \left(\frac{\mathrm{F}_\pi{}^2}{8} + \frac{1}{2e^2}F'^2\right) \qquad \text{and} \qquad C_2 = 2\pi\int_0^\infty dr\ \frac{1}{2e^2}\frac{\sin^4 F}{r^2}\ . \tag{A.6}$$

As pointed out by Kurihara et al. [14], if one uses the form $\Theta = \theta$ as done in Ref.[13] this equation implies

$$C_1(1-n^2)\cos\theta = 0\ . \tag{A.7}$$

which can only be satisfied for $n=\pm 1$. Therefore, for $n\neq\pm 1$ there is a local unstability. Instead of solving Eq.(A.5) numerically Kurihara *et al.* proposed to use the trial function defined in Eq.(14).

Finally, the equation for the chiral angle F is

$$\left(\frac{\mathrm{F_\pi}^2}{4}r^2+\frac{\alpha_1}{e^2}\sin^2 F\right)\ F'' \ + \ \frac{\mathrm{F_\pi}^2}{2}\ r\ F' - \frac{\mathrm{F_\pi}^2}{4}\alpha_1 \sin F\cos F + \frac{\alpha_1}{e^2}F'^2 \sin F\cos F$$
$$- 2\frac{\alpha_2}{e^2}\frac{\sin^3 F\cos F}{r^2} - \frac{m_\pi{}^2\mathrm{F_\pi}^2}{4}r^2\sin F = 0\ , \tag{A.8}$$

with the boundary conditions $F(0)=\pi$ and $F(\infty)=0$. The explicit expressions of α_1 and α_2 have been given in Eq.(22).

The use of the collective coordinate method for the quantization of the $SU(2)$ Lagrangian leads to the four last terms of Eq.(28). The explicit form of the moments of inertia $\mathcal{I}_i$ appearing in such an equation is

$$\mathcal{I}_i \ = \ \int_0^\infty \ dr\ r^2\sin^2 F\left[\left(\frac{\mathrm{F_\pi}^2}{4}\ + \ \frac{1}{e^2}F'^2\right)\zeta_i \ + \ \frac{1}{e^2}\frac{\sin^2 F}{r^2}\eta_i\right]\ , \tag{A.9}$$

where the coefficients ζ_i and η_i are given by

$$\begin{array}{ll}
\zeta_1 = \pi\int_0^\pi\ d\theta\ \sin\theta\left(\Theta'^2+n^2\frac{\sin^2\Theta}{\tan^2\theta}\right)\ , & \eta_1 = n^2\pi\int_0^\pi\ d\theta\ \sin\theta(1+\cos^2\theta)\Theta'^2\ \frac{\sin^2\Theta}{\sin^2\theta}\ , \\
\zeta_2 = \pi\int_0^\pi\ d\theta\ \sin\theta(1+\cos^2\Theta) & \eta_2 = \pi\int_0^\pi\ d\theta\ \sin\theta\left(\Theta'^2\cos^2\Theta+n^2\frac{\sin^2\Theta}{\sin^2\theta}\right)\ , \\
\zeta_3 = 2\pi\int_0^\pi\ d\theta\ \sin\theta\ \sin^2\Theta & \eta_3 = 2\pi\int_0^\pi\ d\theta\ \sin\theta\ \Theta'^2\sin^2\Theta\ , \\
\zeta_4 = \frac{8}{3}\pi\ , & \eta_4 = \frac{8}{3}\pi\ .
\end{array} \tag{A.10}$$

Appendix B: Quantization rules and the dibaryon isospin

In case the background field is restricted to $SU(2)$ one finds that in the bound state approach (as in the case of the normal $SU(2)$ Skyrme model) there is a quantization ambiguity on whether a baryon number B configuration has to be quantized as a fermion or as a boson. Furthermore one has no information on which $SU(3)$ multiplet a quantized bound state system with J, I and S should belong. In order to avoid these ambiguities in the bound state approach the authors of Ref.[16] suggested to introduce a third light flavour - degenerated with the u and d flavours - called “funny strange” flavour and to embed the bound state ansatz (4) into $SU(4)^{(f)}$ (where the label (f) distinguishes this group from a physical $SU(4)$ flavour group) as follows:

$$U^{(f)} = \left(\begin{array}{cc}\sqrt{U_\pi^{(f)}} & 0\\ 0 & 1\end{array}\right)\ U_K^{(f)}\ \left(\begin{array}{cc}\sqrt{U_\pi^{(f)}} & 0\\ 0 & 1\end{array}\right)\ . \tag{B.1}$$

Now $U_\pi^{(f)}$ belongs to $SU(3)^{(f)}$,

$$U_\pi^{(f)} = \left(\begin{array}{cc}u_\pi & 0\\ 0 & 1\end{array}\right)\ , \tag{B.2}$$

and $U_K^{(f)}$ has in terms of $K^{(f)}$ the same form as U_K in terms of K, where

$$K^{(f)} = \begin{pmatrix} K \\ 0 \end{pmatrix} = \begin{pmatrix} K^+ \\ K^0 \\ 0 \end{pmatrix} . \tag{B.3}$$

After collective quantization of the $SU(3)^{(f)}$ flavour degrees of freedom (u, d and f) the Wess-Zumino term [5] leads to a constraint on the "funny" right-hypercharge $Y_R^{(f)}$ [16],

$$Y_R^{(f)} = \frac{N_c\ B + S}{3} , \tag{B.4}$$

where N_c is the number of colours, B the baryon charge of the configuration and S the physical (*not* "funny") strangeness. The derivation of this relation assumes only that the background field u_π belongs to $SU(2)$ and that K has the form (6). The result (B.4) is as well applicable for the $SU(3)^{(f)}$–extended axial rotor system with $n \geq 2$ where $n = B$. For $B = 2$ and $N_c = 3$ we have therefore the constraint

$$Y_R^{(f)} = 2 + \frac{S}{3} . \tag{B.5}$$

Of all possible $SU(3)^{(f)}$ multiplets with $Y_R^{(f)}$ given by Eq.(B.5) only the "minimal ones" (which are naturally the energetically lowest ones) will interest us. "Minimal" means that they are composed in a minimal way out of p $SU(3)^{(f)}$ triplet and q antitriplet representations. Therefore they have

$$Y_R^{(f)} = \frac{p + 2q}{3} \qquad \text{and} \qquad I_R^{(f)} = \frac{p}{2} , \tag{B.6}$$

where $I_R^{(f)}$ is the right isospin.

The multiplets are then described by the $SU(3)^{(f)}$ rotation matrices

$$D^{p,q}_{Y^{(f)} I^{(f)} I_3^{(f)}, Y_R^{(f)} I_R^{(f)} I_{R,z}^{(f)}} (\gamma_1, \gamma_2, ..., \gamma_8) , \tag{B.7}$$

where $\gamma_1, \gamma_2, \gamma_3$ are of course the physical Euler angles for isospin rotations, $Y^{(f)}$ is the funny hypercharge of a member of the multiplet, $I^{(f)}$ its isospin, $I_3^{(f)}$ the projection of the isospin in the lab. frame and $I_{R,z}^{(f)}$ the projection of the right isospin in the body-fixed frame.

Since the "funny strangeness" was introduced only to serve as an auxiliary quantity, the physical states should belong to those $SU(2)$ submultiplets of a given $SU(3)^{(f)}$ multiplet which do not have any "funny strange" component. In practice this means that we only work with the upper row of a given minimal $SU(3)^{(f)}$ multiplet. Thus we have $Y = Y^{(f)} = Y_R^{(f)}$ and $I = I^{(f)} = I_R^{(f)} = p/2$ where Y and I are the hypercharge and the

[5]The derivation of this result is in complete analogy to the one for the quantization of an $SU(3)$ skyrmion. In that case, however, the Wess–Zumino term leads to the constraint $Y_R = N_c\ B/3$ [7].

isospin of the physical states. In this case, the $SU(3)^{(f)}$ rotation matrix (B.7) simplifies to

$$D^{p,q}_{Y_R I I_3, Y_R I I_3^{bf}}\left(\gamma_1,\gamma_2,...,\gamma_8\right)\ , \tag{B.8}$$

which is equivalent, as the isospin content is concerned, to the usual $SU(2)$ isospin rotation matrix

$$D^{I}_{I_3,I_3^{bf}}\ ,(\gamma_1,\gamma_2,\gamma_3) \tag{B.9}$$

where I_3 is the isospin projection in the lab. frame and I_3^{bf} the one in the body-fixed frame.

From Eqs.(B.6,B.5) we can now construct the allowed funny $SU(3)^{(f)}$ multiplets for a bound state configuration with $B = 2$ and given S. For $S = 0$ we find $(p,q) =$ (0,3), (2,2), (4,1) and (6,0) which correspond to the usual 6 quark flavour multiplets: $\bar{1}\bar{0}$, 27, 35 and 28. For $S = -1$, we find $(p,q) = (1,2)$, $(3,1)$ and $(5,0)$ which correspond to the $SU(3)^{(f)}$ multiplets: $\bar{1}\bar{5}$, 24 and 21 respectively. Note that these numbers are the same as for the 5 quark $SU(3)$ multiplets which follow after removal of one quark from the 6 quark system.

In Table 5 all minimal "funny" multiplets (p,q) for the $B = 2$ bound state configurations with strangeness S between 0 and -6 are listed. From that table one can deduce that the states characterized by the rotation matrix of the background field (B.8) correspond in the quark model to those minimal quark flavour configurations which involve only the u and d quarks of a given $B = 2$ state. E.g. for $S = -6$ the funny multiplet is a singlet signalling that there is no u, d content. The role of the strange quarks (6 in the $S = -6$ system) is taken over by the kaons coupled to the $n = 2$ soliton background. Note that any $B = 2$ state with $S \leq -7$ or $S \geq 1$ corresponds to a non-minimal $SU(3)^{(f)}$ multiplet for the rotor.

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Table captions

Table 1: The allowed quantum numbers in the $B = 2$ bound statesystem. In the rows the following quantities are listed: (1) the strangeness S, (2) the allowed values of isospin I, (3) the magnitude of the isospin projection in the body fixed frame, $|I_3^{bf}| \leq I$, (4) the magnitude of the projection of the kaon grand-spin on the body fixed frame, $|T_3| \leq |S/2|$, (5) the magnitude of the projection of the total angular momentum on the body fixed frame, $|J_3^{bf}| = 2|I_3 + T_3|$ (see Eq.(38)) with $|J_3^{bf}| \leq 2$, the parity P as given by Eq.(67), and the number of states with the same J_3^{bf} quantum number, (6) the total angular momentum $J \leq 2$ and the corresponding $SU(3)$ representations.

Table 2: Numerical values of the parameters appearing in the dibaryon mass formulae Eqs.(68-71). SET A corresponds to the massless pion case while SET B corresponds to massive pions.

Table 3: Rotational contributions to the dibaryon masses. Listed are those corresponding to the allowed states with $M_{rot} \leq 250\ MeV$. SET A and B are as in Table 2. NN, $N\Lambda$ and $\Lambda\Lambda$ stand for the sum of the rotational contributions to the corresponding particles and serve as rotational threshold in each particular channel.

Table 4: Contributions (in MeV) to the mass of the H–particle given relative to twice the corresponding contribution to the Λ mass.

Table 5: Allowed "funny" multiplets (p, q) for the $B = 2$ bound state configurations.

Table 1

S	I	$\lvert I_3^{bf}\rvert$	$\lvert T_3\rvert$	$(\lvert J_3^{bf}\rvert)^P_{\rm deg}$	$J^P(SU(3)$ repr.$)\ \leq 2$
0	0	0	0	0^+_1	$1^+(\bar{10})$
	1	0,1	0	$0^+_1;2^-_1$	$0^+(27);2^\pm(27)$
	2	0,1,2	0	$0^+_1;2^-_1$	$1^+(35);2^-(35)$
	3	0,1,2,3	0	$0^+_1;2^-_1$	$0^+(28);2^\pm(28)$
-1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$0^+_2;2^-_1$	$1^+(\bar{10}),0^+(27);2^\pm(27)$
	$\frac{3}{2}$	$\frac{1}{2},\ \frac{3}{2}$	$\frac{1}{2}$	$0^+_2;2^-_2$	$0^+(27),1^+(35);2^\pm(27),2^-(35)$
	$\frac{5}{2}$	$\frac{1}{2},\ \frac{3}{2},\ \frac{5}{2}$	$\frac{1}{2}$	$0^+_2;2^-_2$	$0^+(28),1^+(35);2^\pm(28),2^-(35)$
-2	0	0	0,1	$0^+_1;2^-_1$	$0^+(27);2^\pm(27)$
	1	0,1	0,1	$0^+_3;2^-_2$	$0^+(27),1^+(35,\bar{10});2^\pm(27),2^-(35)$
	2	0,1,2	0,1	$0^+_3;2^-_3$	$0^+(28,27),1^+(35);2^\pm(28,27),2^-(35)$
-3	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2},\ \frac{3}{2}$	$0^+_2;2^-_2$	$0^+(27),1^+(35);2^\pm(27),2^-(35)$
	$\frac{3}{2}$	$\frac{1}{2},\ \frac{3}{2}$	$\frac{1}{2},\ \frac{3}{2}$	$0^+_4;2^-_3$	$0^+(28,27),1^+(35,\bar{10});2^\pm(28,27),2^-(35)$
-4	0	0	0,1,2	$0^+_1;2^-_1$	$1^+(35);2^-(35)$
	1	0,1	0,1,2	$0^+_3;2^-_3$	$0^+(28,27),1^+(35);2^\pm(28,27),2^-(35)$
-5	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2},\ \frac{3}{2},\ \frac{5}{2}$	$0^+_2;2^-_2$	$0^+(28),1^+(35);2^\pm(28),2^-(35)$
-6	0	0	0,1,2,3	$0^+_1;2^-_1$	$0^+(28);2^\pm(28)$

Table 2

	M_{sol}	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	ε	c_1	c_2
	MeV	fm	fm	fm	MeV		
SET A	1675	2.62	1.75	1.17	238	.623	.436
SET B	1675	3.22	2.11	1.42	226	.554	.334

Table 3

	(I,J^π)	This model SET A	This model SET B	Ref.[12]
S=0	$0,1^+$	75	61	76
	$1,0^+$	113	93	120
	NN	147	147	147
	$1,2^-$	216	177	212
$S=-1$	$1/2,0^+$	61	46	76
	$1/2,1^+$	87	75	83
	NΛ	92	85	90
	$3/2,0^+$	157	138	160
	$1/2,2^-$	165	129	169
$S=-2$	$0,0^+$	21	11	39
	ΛΛ	37	22	33
	$1,0^+$	79	66	88
	$1,1^+$	107	98	115
	$0,2^-$	119	88	130
	$1,2^-$	187	152	182
	$0,2^+$	247	195	higher

Table 4

	SET A	SET B
ΔM_{sol}	-52	-54
$2\Delta\varepsilon$	32	32
ΔM_{rot}	-15	-12
$M(H)-2M(\Lambda)$	-34	-34

Table 5

S	$Y_R^{(f)}$	p	q	$SU(3)^{(f)}$ repres.	$I = I_R$
0	2	0	3	$\bar{10}$	0
		2	2	27	1
		4	1	35	2
		6	0	28	3
-1	5/3	1	2	$\bar{15}$	1/2
		3	1	24	3/2
		5	0	21	5/2
-2	4/3	0	2	$\bar{6}$	0
		2	1	15	1
		4	0	15	2
-3	1	1	1	8	1/2
		3	0	10	3/2
-4	2/3	0	1	$\bar{3}$	0
		2	0	6	1
-5	1/3	1	0	3	1/2
-6	0	0	0	1	0